

Unified Description of Plausible Cause and Effect of the Big Bang

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Plausible cause and effect of the big bang model are presented without violating conventional laws of physics. The initial cosmological singularity is resolved by introducing the uncertainty principle of quantum theory. We postulate that, preceding the big bang at the end of the gravitational collapse, the total observed mass including all forms of energy of the universe condensed into the primordial black hole (PBH) in a state of isotropic and minimal chaos (i.e., nearest to the absolute zero temperature). The frozen energy of the collapsed state is quantized such that each quantum of frozen energy (the cosmic particle) is characterized by the fundamental constants of the general theory of relativity and of quantum theory. The minimum size and minimum lifetime of the cosmic particle are estimated within the framework of the uncertainty principle. It is considered that these cosmic particles are bosons with intrinsic spin zero. The minimum size and thermodynamic properties of the PBH are estimated within the framework of the Bose-Einstein statistics by taking into account the total observed mass of the universe. The mechanism of the big bang is given and the currently accepted Planck temperature at the beginning of the big bang is predicted. The critical mass density, the mean mass density, and the missing mass are calculated as a function of the Hubble time of expansion. Our calculations show that the universe is closed but expanding. However, the universe remains inside the event horizon of the primordial black hole.

Subj- class: cosmology: theory - pre- big bang physics; post- big bang effect.

1. INTRODUCTION

Our current understanding of the evolution of the universe is primarily based on the standard big bang model, an explosion at Planck temperature $\sim 10^{32}$ K (Planck 1899). The important observational evidences in support of this model are: (i) The universe is undergoing an isotropic homogeneous expansion, a fact first established by Hubble (1929). This has been experimentally verified in almost all the studies of thousands of galaxies across the universe with minimal uncertainty; (ii) Since the early 1940's nuclear physics began to play a significant role in cosmology when it was realized (Gamow 1946, 1948; Alpher, Bethe, & Gamow 1948; Alpher, Follin, & Herman 1953) that if our universe began with extremely high densities and temperatures in excess of 10^{12} K, nucleosynthesis would have occurred. The calculated primordial abundances of the light elements D, ^3He , ^4He , and ^7Li , in the early universe, within the framework of nuclear reactions and the standard model of particle physics, are in strikingly good agreement with the observed data (Boesgaard & Steigman 1985); (iii) The presence of the cosmic microwave background radiation (CMBR), first discovered by Penzias & Wilson (1965); followed by the most spectacular confirmation of its existence which came from the observation of the 2.72-K CMBR of NASA's Cosmic Background Explorer (COBE) Satellite, in the early 1990s (e.g., Mather et al. 1990, 1994; Kogut et al. 1992; Fixsen et al. 1994). These observations confirming the big bang evolution of the universe are consistent with the predictions of the standard big bang model which represents the physically acceptable solutions (Friedmann 1922; Robertson 1935; Walker 1936; Weinberg 1972, 1993; Kolb & Turner 1994) of Einstein's equations of the general theory of relativity. Seemingly, the standard big bang model is superior to currently

popular theories such as superstring theory (Greene 2000) and the inflationary cosmological model (Guth 1997) in accounting for the observational data associated with the physical universe. Nevertheless, the occurrence of cosmological singularity preceding the big bang in the framework of the general theory of relativity, as shown by the theorems of Penrose (1965), and of Hawking & Penrose (1970), raises some of the deepest questions concerning the initial singularity, a state of infinite density and temperature.

The problem of cosmological singularity arises because we are apparently forced to choose only one of the two mutually complementary fundamental theories of nature. One is Einstein's general theory of relativity based on the classical concept of continuous space-time mode of the description of nature, the other is quantum theory based on Planck's discovery (1899) of the universal quantum of action (h), which introduces discontinuities into the law of nature, contrary to the fundamental principle of classical physics. The general theory of relativity provides fundamental aspects of our understanding of natural phenomena in the domain of the large-scale structure of the universe and quantum theory in the domain of the microscopic structure of the matter. These two theories, pillars of modern physics, hitherto have been irreconcilable because of the basic reasons presented above. Consequently, to explore the physics of the big bang and to resolve the problem of the initial cosmological singularity, the unphysical state of the infinite density and temperature of the general theory of relativity, we present a physically acceptable state of finite density and temperature preceding the big bang. This is achieved by taking into account the effects of the uncertainty principle of quantum theory because of its significant role in the domain of infinitesimally small region of space-time. Therefore it is proposed that at the end of the gravitational collapse of the universe, without violating the uncertainty principle, there remained an immensely large amount of cold condensed energy of finite density and temperature around the initial singularity preceding the big bang in a spherically symmetrical state, which we call the primordial black hole (PBH). In other words, the primordial black hole is the modified initial singularity preceding the big bang.

In Sect.2, we present the basic assumptions, and the method of quantization of the cold condensed energy of the PBH, based on the principle of dimensional analysis and the reciprocity relation, which is analogous to that of the unitarity condition of quantum theory. In Sect.3, we present properties of the PBH in three parts: 3.1, properties of the quantized energy (i.e., the cosmic particle); 3.2, estimation of the minimum size of the modified initial singularity (i.e., PBH); and 3.3, thermodynamic properties of the (PBH). In Sect.4, a plausible mechanism of the big bang, an explosion, analogous to that of a nuclear explosion, is offered. In Sect.5, we present the results of our calculations for critical mass density, mean mass density, missing mass as a function of the Hubble time of expansion (i.e., \approx the age of the universe) based on our initial conditions. Furthermore we show that the universe is expanding and closes at the boundary of the event horizon of the PBH and presumably will recollapse. Finally in Sect.6, a summary and conclusions are given.

2. QUANTIZATION OF ENERGY

We postulate that during the process of the gravitational collapse of the universe, in the domain of the infinitesimally small region of space-time around the initial singularity, the general theory of relativity breaks down because such a small region belongs to the realm of quantum theory. Consequently, without violating the uncertainty principle, there remained an immensely large and finite density of cold condensed energy (PBH) in a spherically symmetrical state. According to Hawking (1988), "the higher the mass of the black hole, the lower the temperature (p.105); a black hole with a mass a few times that of the sun would have a temperature only one ten millionth of a degree above absolute zero" (p.108). Therefore, even though we cannot know the internal temperature of the black hole, we believe that it is reasonable to postulate that the temperature of the PBH (collapsed state of the universe) with a mass of $\approx 10^{24}$ times that of the sun would have had a temperature of \sim absolute zero. Our reasoning for this postulate is consistent with Penrose's argument based on thermodynamic considerations that the universe was initially very regular (i.e.,

in a state of zero entropy); Chandrasekhar (1990) likewise agreed with Penrose's view, contrary to those who hold that the chaos at the initial singularity was maximal.

The energy of the collapsed state of the universe (PBH) is quantized in such a manner that each quantum of mass (frozen energy) is characterized by the fundamental constants of the general theory of relativity and of quantum theory (namely G, \hbar, c). For this purpose the principle of dimensional analysis and the reciprocity relation, analogous to that of the unitarity condition of quantum theory, are incorporated into our method of quantization so that their physical properties must be independent of the units in which they are measured and also must have lowest value for cosmic unit of mass. Since the dimensions of G, \hbar, c and mass (M) are known, the only simplest dimensionless combinations which can be formed of them are $GM^2/\hbar c$ and $\hbar c/GM^2$. Therefore they must satisfy the following condition:

$$\frac{\hbar c}{GM^2} = \frac{GM^2}{\hbar c}.$$

The only physical solution out of the four solutions $\pm(\hbar c/G)^{1/2}$ and $\pm i(\hbar c/G)^{1/2}$ for M of the above relation, we have chosen the real positive value for the mass M :

$$M = \left(\frac{\hbar c}{G}\right)^{1/2} \cong 2.2 \times 10^{-5} \text{ gm.}$$

This is the *lowest* mass of the quantized frozen energy which we call the cosmic particle. The magnitude of this mass is identical to that of Planck's mass which he derived from his theory of black body radiation (Planck 1899). Although the values of their masses are the same, their physical properties are entirely different. One is an elementary quantum of frozen energy which has only rest mass while the other is the largest quantum of radiation energy which has zero rest mass. The intrinsic spin of this cosmic particle is considered to be zero because: (i) The method of quantization of frozen energy is independent of any spin; (ii) The method is purely classical and the intrinsic spin of any particle is an internal degree of freedom which has no classical analogue; (iii) If this particle had an intrinsic spin, it would not have the lowest cosmic unit of mass, rather its lowest mass would be increased by an amount equivalent to the rotational energy of its intrinsic spin; and finally (iv) Its shape must be perfectly spherical because it is in a state of absolute zero temperature and lowest energy state. Therefore, we conclude that most likely its intrinsic spin is zero and it is a boson.

3. PROPERTIES OF PRIMORDIAL BLACK HOLE

3.1 Properties of quantized particles

The minimum size and minimum lifetime of the quantized cosmic particles, each of mass, $M = (\hbar c/G)^{1/2}$, are determined within the framework of the uncertainty principle (Heisenberg 1927). According to this principle, it is impossible to determine precisely and simultaneously the values of any pair of canonically conjugate pairs (q_r, p_s) pertaining to a system and satisfying the usual commutation rules

$$[q_r, p_r] = i\hbar; [q_r, p_s] = 0 \text{ for } r \neq s; [q_r, q_s] = 0; \text{ and } [p_r, p_s] = 0.$$

In quantitative terms this principle states that the order of magnitude of the product of uncertainties in the measurement of two variables must be at least Planck's constant h divided by 2π ($\hbar = h/2\pi$). Therefore, we should have

$$\Delta q \cdot \Delta p \geq \hbar,$$

where p and q are canonically conjugate to each other in the classical Hamiltonian sense. Profound implications of the uncertainty principle in physical terms are presented in complementarity principle

(Bohr 1928). In a four-dimensional space-time, coordinates of a particle and its corresponding components of energy-momentum, the uncertainty relations are:

$$\Delta x \cdot \Delta p_x \geq \hbar; \quad \Delta y \cdot \Delta p_y \geq \hbar; \quad \Delta z \cdot \Delta p_z \geq \hbar \quad \text{and} \quad \Delta t \cdot \Delta E \geq \hbar.$$

$$\text{Let } (\Delta x)_{min} = (\Delta y)_{min} = (\Delta z)_{min} \equiv L_{min},$$

$$(\Delta p_x)_{max} = (\Delta p_y)_{max} = (\Delta p_z)_{max} \equiv P_{max} = Mc,$$

$$(\Delta t)_{min} \equiv \tau_m \quad \text{and} \quad (\Delta E)_{max} \equiv E = Mc^2,$$

therefore:

$$L_{min} \cong \frac{\hbar}{Mc} = \left(\frac{\hbar G}{c^3} \right)^{1/2} \approx 1.6 \times 10^{-33} \text{ cm and}$$

$$\tau_m \cong \left(\frac{\hbar}{Mc^2} \right) = \left(\frac{\hbar G}{c^5} \right)^{1/2} \approx 5.3 \times 10^{-44} \text{ sec.}$$

$L_{min}(L)$ and τ_m are interpreted as the minimum size of localization and minimum lifetime (time scale) of the quantized cosmic particle. It is significant that the values of Planck's mass, length, and time (Planck 1899) are identical to those of the mass, size, and minimum lifetime of the cosmic particle obtained in the present investigation. However, we demonstrate later in this article, that they are entirely different in their properties and these different physical properties of the cosmic particle play the most important role in the big bang theory. Thus our results provide new significance into Planck's system of units and show that these units are fundamental not only in the theory of black body radiation, in the theory of unification of fundamental interactions, and in the theory of superstrings, but also in the theory of the big bang origin of the universe.

3.2 Structure of the modified initial singularity (PBH)

The constituents of the modified initial cosmological singularity (PBH) are cosmic particles, each of mass, $M = (\hbar c/G)^{1/2}$, and minimum size of localization, $L_{min} = (G\hbar/c^3)^{1/2}$. These particles are bosons and obey the Bose-Einstein statistics (Bose 24; Einstein 1924, 1925a). Consequently, without violating the uncertainty principle of quantum theory and putting all the Bose-Einstein particles in the lowest energy state as $T \rightarrow 0$, the maximum density of matter, ρ_{max} , in the Primordial black hole (PBH) is given as a function of G, \hbar , and c :

$$\rho_{max} = \frac{3}{4\pi} \left(\frac{c^5}{\hbar G^2} \right) \cong 1.2 \times 10^{93} \text{ gm/cm}^3.$$

This result is based on the assumption that each particle has a hard repulsive core of radius $\sim 10^{-33}$ cm, consistent with the uncertainty principle, surrounded by an attractive gravitational force.

The minimum volume and the minimum size (r_{min}) of the structure of the modified initial singularity are determined taking the total mass of the universe to be about 5.68×10^{56} gms: This estimation of mass is based on a typical cosmological model without cosmological constant, compatible with astronomical observations and with Einstein's conception of cosmology (Misner, Thorne, & Wheeler 1973). We have already estimated the mass density to be about 10^{93} gm/cm³ which together with the total mass of the universe leads to the minimum volume 4.76×10^{-37} cm³ and the minimum size of the radius (r_{min}) 4.8×10^{-13} cm, about one hundred thousandth of the size of an atom. These results are significant because they provide a physical interpretation of the initial cosmological singularity predicted by the general theory of relativity. It is not a mathematical point but it has a finite structure of the order of 10^{-12} to 10^{-13} cm; and it is not a 'state' of infinite density but of finite density of the order of 10^{93} gm/cm³. Nevertheless, the distance of the order of 10^{-13} cm is remarkably close to singularity and the matter density of the order of 10^{93} gm/cm³ can be considered immensely large and still finite.

3.3 Thermodynamic properties of the PBH

From the available information about the temperatures of white dwarfs, neutron stars, and black holes (Hawking 1988), and as well as from Penrose's argument (Chandrasekhar 1990) we infer, as stated in Sect.2, that the temperature of the PBH goes to zero in its final collapsed state. To calculate the entropy of this system we utilize Boltzmann's relation between the entropy S and thermodynamic probability W :

$$S = k_\beta \ln W + S_\circ$$

where k_β and S_\circ are Boltzmann's and integration constants respectively. The first term, $k_\beta \ln W$ is interpreted as the internal entropy of the matter present inside the PBH and S_\circ as the external entropy but within the boundary of the surface area of the event horizon which is inaccessible to an exterior observer. Since the quantized matter inside the PBH consists of bosons, its properties can be described by Bose-Einstein statistics. Therefore the probability W can be written as (Einstein 1924,1925a):

$$W = \prod_s \frac{(N^s + Z^s - 1)!}{N^s! (Z^s - 1)!}.$$

Where N^s are the number of particles and Z^s are the number subcells in the s-th cell. We have pointed out above, all N bosons go into the lowest energy state as $T \rightarrow 0$, in this limit $N^s = 0$ for all other cells. Therefore $W \rightarrow 1$, $k_\beta \ln W \rightarrow 0$, and $S \rightarrow S_\circ$. Consequently the internal entropy of PBH $\rightarrow 0$ as $T \rightarrow 0$. This result is consistent with the prediction of Einstein in 1925 that B-E gas satisfies the third law of thermodynamics (Einstein 1925b; Pais 1982).

The entropy S_\circ , inaccessible to an exterior observer, is calculated within the framework of the theory of black hole entropy based on the second law of thermodynamics and information theory developed by Bekenstein (1973). Therefore

$$S = S_\circ = \frac{1}{2} \left(\frac{\ln 2}{4\pi} \right) k_\beta c^3 \hbar^{-1} G^{-1} A$$

where A is the surface area of the event horizon of the PBH corresponding to the Schwarzschild's radius $R_{Sch} = 2GM/c^2$ and $A = 16\pi M^2 G^2/c^4$. The estimated value of the entropy of PBH is found to be $\approx 1.32 \times 10^{107} \text{ erg} \cdot K^{-1}$ which is extremely large as expected.

Finally the internal pressure in the final phase of the gravitational collapse (PBH) is evaluated within the framework of correspondence principle (Bohr 1923) and Newton's gravitational theory, assuming that the pressure inside the PBH is generated by an immense, attractive gravitational force alone. At a distance r from the center of PBH, the pressure $P(r)$ is given by:

$$P(r) = \frac{2\pi}{3} G \rho_{max}^2 (r_{min}^2 - r^2); \quad r \leq r_{min},$$

where ρ_{max} and r_{min} are the maximum matter density and minimum radius of the (PBH) respectively. The maximum pressure when $r \rightarrow 0$, at the center, $P(r \rightarrow 0)$ is calculated using the earlier estimated values of $\rho_{max} \approx 1.2 \times 10^{93} \text{ gm/cm}^3$ and $r_{min} \approx 4.8 \times 10^{-13} \text{ cm}$. The result is $P(r \rightarrow 0) \approx 5 \times 10^{154} \text{ dynes/cm}^2$, indeed extremely large.

4. POSSIBLE MECHANISM OF THE BIG BANG

Based on our knowledge of nuclear fission and nuclear explosives, we give a simplified plausible account of the mechanism of the big bang. Just as the salient features of the fission of a nucleus, say uranium by slow neutrons, have been fully interpreted in terms of the liquid drop model, originally by Bohr & Wheeler (1939); subsequently, this information has been successfully utilized in the design of various types of explosives – from nuclear fission to nuclear fusion hydrogen bombs. The basic design of a nuclear implosion bomb (Krane 1987), consists of a solid spherical subcritical

mass of the fissionable material (e.g., ^{235}U or ^{239}Pu) surrounded by a spherical shell of conventional chemical explosives. An initiator at the center provides neutrons to start a chain reaction. When the conventional explosives are detonated in exact synchronization, a spherical shock wave compresses the fissionable material into a supercritical state, resulting in an explosion. Now we illustrate that many of these basic characteristics of a nuclear fission bomb are naturally present in PBH.

In the present instance: (1) the PBH which consists of unstable quantized cosmic particles (bosons) is analogous to the solid spherical subcritical mass of the fissionable material; (2) the vacuum energy fluctuations caused by the uncertainty principle of quantum theory surrounding the PBH (within the boundary of its event horizon) can produce results similar to those of the conventional explosives surrounding the fissionable material of the bomb; (3) the unstable cosmic particles, each of Planck's mass, are analogous to the nuclei of the fissionable material; and (4) the constraint of uncertainty principle and the maximum gravitational pressure at the innermost central region of PBH may trigger the explosion (big bang) analogous to that of the initiator at the center of the bomb which provides neutrons to begin the chain reaction resulting in explosion. In an explosion of a nuclear fission bomb, only a very small fraction of mass of each nucleus of fissionable material is transformed into kinetic energy while in the explosion of the PBH, total mass ($M \cong 2.2 \times 10^{-5}$ gms) of each unstable cosmic particle is converted into kinetic energy resulting in temperature of 10^{32} K, the currently accepted temperature at the beginning of the big bang. The total mass of the PBH consisting of nearly 10^{61} cosmic particles, each of mean lifetime (decay rate) of the order of $\tau_m \approx 5.3 \times 10^{-44}$ sec, is transformed into kinetic energy $\approx 10^{77}$ erg, practically in less than 10^{-40} second resulting in the biggest explosion, the big bang. In the following section, we examine the aftermath of the big bang.

5. THE COSMIC MEAN MASS DENSITY AND THE HUBBLE TIME OF EXPANSION

Our discussions of the cosmic mean mass density and the Hubble time of expansion are based on the assumptions that the universe is homogeneous and isotropic. Further, Einstein's cosmological principle requires that the world view of any observer relative to himself must have spherical symmetry about himself. Therefore let us consider a sphere of galaxies of radius $R(t)$ with the observer at the origin. According to a theorem of Newton which remains valid also in the framework of general theory of relativity as shown by Birkhoff (1927); the total energy E of a particle of mass m on the surface of the sphere of radius $R(t)$ with a uniform mass density $\rho(t)$ is (Weinberg 1972, p.475):

$$\begin{aligned} E &= \frac{1}{2} m \frac{|X(t_o)|^2}{R^2(t_o)} \left[\left(\frac{dR(t)}{dt} \right)^2 - \frac{8\pi G}{3} \rho(t) R^2(t) \right] \\ &= \frac{1}{2} m \frac{|X(t_o)|^2}{R^2(t_o)} R^2 H^2 [1 - \Omega], \end{aligned} \quad (1)$$

where $H = \dot{R}/R$ (where $\dot{R} \equiv dR/dt$), is the Hubble constant which determines the expansion rate of the universe; the total energy, E , remains constant as the universe expands: and Ω is the ratio of the density ρ to the critical density ρ_c :

$$\Omega \equiv \frac{\rho}{\rho_c}; \quad \text{and} \quad \rho_c \equiv \frac{3H^2}{8\pi G}. \quad (2)$$

The equation (1) is equivalent to the following Friedmann equation of the cosmology, a solution of Einstein's equations of the general theory of relativity without the cosmological constant, based on the Robertson-Walker metric (Robertson 1935; Walker 1936; Weinberg 1972; Kolb & Turner 1994)

$$k + \dot{R}^2 = \frac{8\pi G \rho R^2}{3},$$

or in terms of Hubble Constant H and $\Omega(\rho/\rho_c)$:

$$k = -R^2 H^2 [1 - \Omega] \quad (3)$$

Provided the constant energy E and the constant k are given by the relation:

$$E = -\frac{1}{2}m \frac{|X(t_o)|^2}{R^2(t_o)} k. \quad (4)$$

Equation (1), which is equivalent to the Friedmann Eq.(3) of cosmology, is valid for all times. However note that ρ , ρ_c , H , and $\Omega(\rho/\rho_c)$ are not constant but change as the universe expands. We shall now consider three cases of Eq.(1) for the future evolution of our universe: (i) For $k = -1$, Eq.(4) shows that the total energy E is positive-definite, and Eq.(1) shows that the expansion velocity \dot{R} never can go to zero, Ω must be less than one, consequently if the universe is presently expanding it must continue to expand forever (i.e., the universe is open); (ii) For $k = +1$, Eq.(4) shows that the total energy E is negative-definite, and therefore Eq.(1) shows that Ω must always remains greater than one, then at a finite time after the big bang origin of the universe, the universe will achieve a maximum expansion radius R , the expansion velocity \dot{R} goes to zero, and then it will begin to recollapse (i.e., the universe is closed); and finally (iii) For $k = 0$, Eq.(4) shows that the total energy, E , of the cosmic particle is zero, and Eq.(1) for this case shows that $\Omega = 1$, and if the universe is presently expanding it must continue to expand forever since expansion velocity, \dot{R} , asymptotically approaches zero as $\rho(t)R^2(t) \rightarrow 0$ as $R(t) \rightarrow \infty$; assuming $\rho(t)R^3(t)$ is constant which expresses the conservation of mass in the Newtonian considerations. From these results it is evident that the hot big bang model, more properly the Friedmann-Robertson-Walker cosmology (standard model of cosmology) does not give a definite prediction as to how the universe will end, i.e., whether the universe is open or closed. Furthermore, it does not attempt to describe what might have existed preceding the big bang and how it might have “banged”. In essence, the standard cosmological model, although the most successful model of cosmology, describes the evolution of the universe, the aftermath of the big bang, from unknown initial conditions and ends with the unknown final future of the universe.

Therefore it is of major interest to determine the critical mass density, ρ_c , the present mean mass density, ρ_p , and the ratio of the mean density to the critical density, $\Omega_p = \rho_p/\rho_c$, as a function of the Hubble time of expansion based on our postulate that all the matter in the universe, namely $M_p \cong 5.68 \times 10^{56}$ gm (Misner et al. 1973) existed preceding the big bang. We show in the Appendix that the total mass which includes all forms of energy is conserved in the standard model of cosmology. The currently popular accepted value for the Hubble constant, H , based on 15 Kilometers per second per million light year is:

$$H \cong (6.31 \times 10^{17} \text{ sec})^{-1}. \quad (5)$$

Case I. The critical mass density, ρ_c , corresponding to the above value of H ; the Hubble time of expansion, $T_H = H^{-1} \approx 20$ billion years; and the Hubble radius of expansion $R = c/H \approx 1.89 \times 10^{28}$ cm is given:

$$\rho_c = \frac{3H^2}{8\pi G} \approx 4.5 \times 10^{-30} \text{ gm/cm}^3. \quad (6)$$

Case II. Consistent with our postulate that all matter present in the universe, namely $M_p = 5.68 \times 10^{56}$ gm, existed preceding the big bang in the primordial black hole, it must remain the same after the big bang as $R(t)$ varies with time, as required by the law of conservation of mass (see Appendix). Consequently the mean mass density, ρ_p in the present work, corresponding to the Hubble time of expansion $T_H = H^{-1} \approx 20$ billion years and the initial mass, $M_p = 5.68 \times 10^{56}$ gm is:

$$\rho_p = \frac{3M_p}{4\pi c^3 H^{-3}} \approx 2.01 \times 10^{-29} \text{ gm/cm}^3 \quad (7)$$

compared to the critical mass density, ρ_c , for $T_H \approx 20$ billion years is $\approx 4.5 \times 10^{-30}$ gm/cm³. Therefore the universe must be closed, since $\Omega_p = \rho_p/\rho_c > 1$ and it must also be expanding.

Now let us determine when the expansion of the universe stops. For this purpose we are going to prove that when $E = 0$, in Eq.(1) or $k = 0$, in Eq.(3), and $\Omega = \Omega_p = 1$ (i.e., when $\rho_p = \rho_c$), the expansion of the universe stops and closes.

Let H_{min} be the minimum Hubble constant; R_{max} , the maximum radius of expansion; $\rho_{p,min}$, the minimum mass density; and $\rho_{c,min}$, the minimum mean critical mass density, then since $\rho_{p,min} = \rho_{c,min}$, we obtain:

$$\frac{3M_p}{4\pi R_{max}^3} = \frac{3H_{min}^2}{8\pi G};$$

or, equivalently

$$\frac{3M_p}{4\pi R_{max}^3} = \frac{3c^2}{8\pi G R_{max}^2}; \text{ since } H_{min} = \frac{c}{R_{max}}.$$

The above relation gives:

$$R_{max} = \frac{2GM_p}{c^2} = R_{Sch}, \quad (8)$$

where R_{Sch} is the Schwarzschild radius of the primordial black hole (PBH). Consequently, the expansion of the universe must close at the boundary of the event horizon, $R(t) \leq R_{Sch}$, of the PBH after the big bang since nothing can escape from it. This result is consistent with a mathematical proof given by Pathria (1972). Therefore the maximum Hubble time, $T_{H,max}$, of expansion of the universe aftermath of the big bang in the present case is:

$$T_{H,max} = H_{min}^{-1} \leq \frac{R_{Sch}}{c} = \frac{2GM_p}{c^3} \approx 89 \text{ billion years.} \quad (9)$$

Now we discuss the question of mass difference (i.e., the missing mass) as a function of the Hubble time of expansion t , between the total critical mass, $M_c(t)$, and the total initial mass, M_p , which must remain constant, as $R(t)$ varies with time consistent with the law of conservation of mass. Since the Hubble time of expansion, $T_H = H^{-1}(t) = R(t)/c$, the total critical mass, $M_c(t)$, as a function of time t , is given by

$$M_c(t) = \frac{4}{3}\pi R^3(t) \cdot \frac{3H^2(t)}{8\pi G} = \frac{c^3}{2GH(t)}. \quad (10)$$

Therefore the percentage of missing mass as a function of expansion time t , aftermath of the big bang is given by:

$$\frac{[M_p - M_c(t)]}{M_p} 10^2. \quad (11)$$

We now proceed to calculate the critical mass density $\rho_c(t)$, the present mean mass density $\rho_p(t)$, the critical total mass $M_c(t)$, and the percentage of missing mass as a function of the Hubble time of expansion, $T_H = H^{-1}(t)$ by using the Equations (6), (7), (10), and (11) respectively; and also Ω_p , the ratio of the present mean mass density ρ_p to the critical mass density ρ_c . The results of our calculations are given in Table 1. It is clear from the Table 1 that as the universe expands, with increasing value of the Hubble time of expansion, T_H : $\rho_c(t)$, $\rho_p(t)$, $\Omega_p(t)$, and the percentage of missing mass, all of them decrease. However, the total critical mass $M_c(t)$ increases and the initial mass M_p remains the same consistent with the law of conservation of mass-energy. By the time the expansion closes at the boundary, $R(t) \leq R_{Sch}$, of the event horizon of the primordial black hole: $\rho \approx 2.27 \times 10^{-31}$ gm/cm³; $M_c(t) \rightarrow M_p \approx 5.68 \times 10^{56}$ gm; $R(t) \rightarrow \leq R_{Sch} \approx 8.4 \times 10^{28}$ cm; $T_{H,max} \rightarrow \approx 89 \times 10^9$ years; and the missing mass is zero. It is also seen from the Table 1 that corresponding to the currently popular accepted value for the Hubble constant, $H \approx (6.31 \times 10^{17} \text{ sec})^{-1}$ (i.e., the age of the universe, $H^{-1} = T_H \approx 20 \times 10^9$ years): $\rho_c = 4.5 \times 10^{-30}$ gm/cm³; $\rho_p =$

2.01×10^{-29} gm/cm³; $\Omega_p = 4.47$; the total critical mass $M_c = 1.278 \times 10^{56}$ gm compared to the total initial mass $M_p = 5.68 \times 10^{56}$ gm (i.e., the missing mass ≈ 77.70 percentage); and the universe is expanding. However, note that the universe remains inside the event horizon of the primordial black hole (i.e., the modified initial singularity).

TABLE 1

Initial mass of the PBH, $M_p = 5.68 \times 10^{56}$ gm (observed mass)
 $T_H(H^{-1}) = T_o \times 10^9$ years (Hubble time of expansion)
 $\rho_c = \rho_o \times 10^{-28}$ gm/cm³ (critical density)
 $M_c = M_o \times 10^{55}$ gm (corresponds to critical density)
 $\rho_p = \rho \times 10^{-28}$ gm/cm³ (density in the present work)
 $M_p = M \times 10^{56}$ gm (initial mass preceding the big bang)
 $\Omega_p = \rho_p / \rho_c$ [in the present work Ω_p is a function of $T_H(H^{-1})$]

T_o	ρ_o	M_o	ρ	M	Ω_p	Percentage of missing mass
3	2.006	2.01	59.185	5.68	29.50	96.50
5	0.719	3.19	12.78	5.68	17.77	94.40
10	0.1798	6.387	1.598	5.68	8.89	88.75
15	0.0799	9.581	0.473	5.68	5.92	83.13
20	0.045	12.777	0.201	5.68	4.47	77.70
30	0.01997	19.16	0.0591	5.68	2.96	66.30
40	0.01124	25.6	0.02497	5.68	2.22	54.93
50	0.00719	31.93	0.01278	5.68	1.78	43.80
60	0.00499	38.32	0.0074	5.68	1.48	32.50
80	0.00289	51.1	0.0031	5.68	1.08	10.03
89*	0.00227	56.8	0.00227	5.68	1.00	0.00

*89 Billion years corresponds to the Hubble time of expansion for the length of Schwarzschild radius (R_{Sch}/c) event horizon of the PBH.

Note: Initial mass-energy remains constant before and after the big bang. Expansion closed at the boundary of the event horizon. The mass density is interpreted as the total energy divided by c^2 .

SUMMARY AND CONCLUSIONS

The big bang theory poses some of the deepest questions in physics today, for example: (1) what, if anything, preceded the big bang, and how did it happen?; and (2) how will the universe end, i.e., is the universe open, flat, or closed? To search for the most plausible physical solution, we have postulated that during the process of gravitational collapse at the infinitesimally small region of space-time, the general theory of relativity breaks down because such a small region belongs to the realm of quantum theory. Consequently, at the end of the gravitational collapse, instead of the initial cosmological singularity with infinite density and temperature, we now have an extended infinitesimally small structure of cold condensed energy (PBH) with finite density and temperature, preceding the big bang. This result has been achieved by taking into account the effect the uncertainty principle of quantum theory.

The frozen energy of the collapsed state has been quantized in such a manner that each quantum of energy (mass) called cosmic particle, is characterized by the fundamental constants, G , c , \hbar , of

the general theory of relativity and quantum theory. The present method of quantization predicts four solutions, $\pm(\hbar c/G)^{1/2}$ and $\pm i(\hbar c/G)^{1/2}$ for the mass of the cosmic particle with intrinsic spin zero. In the present work we have chosen the real positive value for the mass of the cosmic particle, $M = (\hbar c/G)^{1/2} \cong 2.2 \times 10^{-5}$ gms. We have also determined its minimum size of localization, $L_{min} \cong 1.6 \times 10^{-33}$ cm, and minimum lifetime $t_m \cong 5.3 \times 10^{-44}$ sec within the framework of Heisenberg's uncertainty principle. It is significant that the values of Planck's system of units: mass, length, and time, derived by Planck (1899), when neither theory of relativity nor quantum theory had been discovered, are identical to those obtained in the present work within the framework of these two fundamental theories of the twentieth century. However, they have different physical properties: In Planck's work, mass $M_p \equiv h\nu_{max}/c^2$ is kinetic energy with rest the mass zero and, ν_{max} represents the largest frequency of his classic theory of black body radiation; length $L_p \equiv \lambda_{min}$, λ_{min} represents the minimum wavelength of the radiation; and time $T_p \equiv 1/\nu_{max}$, the period of vibration of wavelength λ_{min} (i.e., the minimum time scale). In the present investigation, M represents the rest mass of the quantized cosmic particle; length $L(L_{min})$ represents the minimum size of localization of the cosmic particle; and time $T(\tau_m)$ represents the minimum lifetime (i.e., the minimum time scale) of the cosmic particle.

The above properties of the cosmic particles (bosons), constituents of the extended structure of the cosmological singularity (PBH); have been utilized to determine the physical properties of the modified initial singularity (PBH), namely: (1) the maximum matter density, $\rho_{max} \cong 1.2 \times 10^{93}$ gms/cm³; (2) the minimum volume $\cong 4.76 \times 10^{-37}$ cm³; (3) the minimum radius, $r_{min} \cong 4.8 \times 10^{-13}$ cm; (4) the entropy, $S \cong 1.37 \times 10^{107}$ erg.K⁻¹; and (5) the maximum gravitational pressure at the center, $P(r \rightarrow 0) \approx 5 \times 10^{154}$ dynes/cm², of the PBH in its final phase of the gravitational collapse state. Based on the properties of the cosmic particles and those of the PBH, we have presented in Sect.4 a most reasonable account of the mechanism of the explosion of the matter of the PBH, resulting in a temperature of 10^{32} K at the beginning of the big bang and energy evolved $\approx 10^{77}$ erg.

Furthermore, we have calculated the cosmic mean mass density $\rho_p(t)$, the critical mass density $\rho_c(t)$, the ratio of the mean mass density to the critical mass density Ω_p , and the missing mass as a function of the Hubble time of expansion $T_H = H^{-1}(t)$, based on our postulate that all the matter present in the universe, namely $M_p = 5.68 \times 10^{56}$ gm (Misner et al. 1973), existed preceding the big bang inside the primordial black hole (PBH). The results of our calculations are presented in Table 1. It is clear from the Table 1, that as the universe expands, with increasing value of the Hubble time $T_H = H^{-1}(t)$: $\rho_c(t)$, $\rho_p(t)$, $\Omega_p(t)$, and the missing mass decrease; whereas the total critical mass $M_c(t)$ increases and the initial mass M_p remains the same as required by the law of conservation of mass (energy). It is relevant to point out here that the percentage of missing mass given in Table 1, is the percentage of mass difference between the initial mass M_p and the total critical mass $M_c(t)$ as explained in Sect.5, Eq.(11). Consequently, if the total observed visible mass density $\rho_{vd}(t) < \rho_c(t)$, then the percentage of missing mass given in Table 1 will be further increased. It should be also noted that $\rho_c(t)$ is independent of the initial mass M_p (i.e., the initial condition) and depends only on the Hubble time of expansion whereas $\rho_p(t)$ depends both on the Hubble time of expansion and the initial mass M_p .

Furthermore, it is seen from Table 1 that corresponding to the currently popular accepted value for the Hubble constant, $H \cong (6.31 \times 10^{-17} \text{ sec})^{-1}$ (i.e., the age of the universe $H^{-1} = T_H \approx 20 \times 10^9$ years): $\rho_c = 4.5 \times 10^{-30}$ gm/cm³; $\rho_p = 2.01 \times 10^{-29}$ gm/cm³; $\Omega_p = 4.56$; the total critical mass $M_c = 1.278 \times 10^{56}$ gm compared to the initial mass $M_p = 5.68 \times 10^{56}$ gm (i.e., the missing mass ≈ 77.70 percentage); and the universe is expanding. Finally, we have shown in Sect.5, that the universe closes at $R(t) \rightarrow \leq R_{Sch}$, Schwarzschild radius of the event horizon of the primordial black hole and presumably will recollapse (i.e., the universe always remains inside the event horizon of the PBH). Unquestionably, our results and inferences are significantly different from those of the predictions of the Friedmann-Robertson-Walker model of cosmology (also known as standard model of cosmology) (Friedmann 1922; Robertson 1935; Walker 1936; Weinberg 1972; Kolb & Turner 1994).

It is not surprising because the standard cosmological model, although the most successful model of modern cosmology, describes the evolution of the universe, aftermath of the big bang, from unknown initial conditions and ends with the unknown final future of the universe as expected from the laws of classical and quantum theories.

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APPENDIX. CONSERVATION OF MASS (ENERGY) IN STANDARD MODEL

Friedmann's equation of energy conservation in Robertson-Walker metric of Einstein's equations of general relativity (Weinberg 1972), 472, equation (15.1.21) is,

$$\frac{dp}{dt}R^3 = \frac{d}{dt} \{R^3(\rho + p)\};$$

or, equivalently

$$d\left(\frac{4}{3}\pi R^3\rho\right) = -pd\left(\frac{4}{3}\pi R^3\right);$$

or,

$$dM = -pdV. \tag{A1}$$

Since in Friedmann's equation of energy conservation, $c = 1$, $U \equiv E = Mc^2$, and according to the Equipartition theorem, $U = E = k_\beta T$, where k_β is the Boltzmann's constant and T is the temperature, therefore equation (A1) can also be written as,

$$dM + \frac{p}{c^2}dV = 0 \tag{A2}$$

$$dU + pdV = 0 \tag{A3}$$

$$k_\beta dT + pdV = 0. \tag{A4}$$

Note that equation (A3) is the familiar form of the first law of thermodynamics and it represents the principle of conservation of energy for an isolated system: dU represents the change in internal energy of the system and pdV represents the work done by the system. They are equal in magnitude and opposite in sign.

The above equations demonstrate that during the process of the evolution of the universe, decrease of mass or energy or temperature is compensated by the increase in volume due to expansion of the universe and thus energy is conserved.

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